A Recursive Decomposition Method for Large Scale Continuous Optimization – Supplementary Material

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1 Detailed Grouping Matrix of RDG

The detailed grouping matrix of the RDG method when used to decompose some of the selected CEC'2010 benchmark problems are presented in Table 1. In Table 1, the rows indicate the groups formed by the RDG method and the columns represent the permutation groups from which the decision variables in each group were extracted. The experimental results show that RDG can extract the interacting decision variables from each group correctly on these benchmark problems.

2 Detailed Decomposition Results of DG2 and FII

The detailed results of the RDG (with $\alpha = 10^{-12}$), DG2 [1], and FII (with $\epsilon = 10^{-2}$) [2] methods when used to decompose the CEC'2010 and CEC'2013 benchmark problems are presented in Table 2. DG2 is a non-parametric method. RDG approximates the threshold values based on the magnitude of the objective space. To test the sensitivity of RDG to the control coefficient α, we also present the decomposition results from RDG with $\alpha = 10^{-10}$ in the table. The FII method uses a fix threshold $\epsilon = 10^{-2}$ in the original paper, which is not sufficient to identify variable interactions in all of the benchmark problems. For a fair comparison, we present the decomposition results from FII which uses the same threshold as RDG (with $\alpha = 10^{-12}$). The experimental results show that the average number of FEs used by RDG is less than that used by DG2 and FII.

3 Detailed Optimization Results from the DECC and CMAESCC Comparisons

The detailed optimization results of the eight decomposition methods when embedded into the DECC and CMAESCC frameworks to solve the CEC'2010 and CEC'2013 benchmark problems are presented in Table 3, Table 4, Table 5 and Table 6. The RDG method achieves overall the best solution quality when compared against the other seven decomposition methods.

References

[1] Mohammad Nabi Omidvar, Ming Yang, Yi Mei, Xiaodong Li, and Xin Yao. DG2: A faster and more accurate differential grouping for large-scale black-box optimization. *IEEE Transactions on Evolutionary Computation*, 2017.

[2] Xiao-Min Hu, Fei-Long He, Wei-Neng Chen, and Jun Zhang. Cooperation coevolution with fast interdependency identification for large scale optimization. *Information Sciences*, 381:142–160, 2017.

Table 1: Detailed grouping matrix of RDG on some selected CEC'2010 benchmark problems. The rows indicate the groups formed by the recursive differential grouping algorithm and the columns represent the permutation groups from which the variables in each group were extracted.

Func	Groups	group size	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{17}	P_{18}	P_{19}	P_{20}
f_4	G01	50	50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
f_5	G01	50	50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
f_7	G01	50	50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
f_8	G01	50	50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
f_9	G01	50	0	0	0	0	0	0	50	0	0	0	0	0	0	0	0	0	0	0	0	0
	G02	50	0	0	0	50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G03	50	0	50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G04	50	0	0	0	0	0	0	0	0	0	50	0	0	0	0	0	0	0	0	0	0
	G05	50	0	0	0	0	50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G06	50	0	0	0	0	0	50	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G07	50	0	0	50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G08	50	0	0	0	0	0	0	0	0	50	0	0	0	0	0	0	0	0	0	0	0
	G09	50	50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G10	50	0	0	0	0	0	0	0	50	0	0	0	0	0	0	0	0	0	0	0	0
f_{14}	G01	50	0	0	50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G02	50	0	0	0	50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G03	50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	50	0	0	0	0
	G04	50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	50	0	0	0
	G05	50	0	0	0	0	0	0	0	0	0	0	0	0	50	0	0	0	0	0	0	0
	G06	50	0	50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G07	50	0	0	0	0	50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G08	50	0	0	0	0	0	0	0	0	0	0	0	0	0	50	0	0	0	0	0	0
	G09	50	50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G10	50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	50	0	0	0	0	0
	G11	50	0	0	0	0	0	0	0	0	0	0	0	50	0	0	0	0	0	0	0	0
	G12	50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	50	0
	G13	50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	50	0	0
	G14	50	0	0	0	0	0	50	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G15	50	0	0	0	0	0	0	0	0	0	0	50	0	0	0	0	0	0	0	0	0
	G16	50	0	0	0	0	0	0	0	50	0	0	0	0	0	0	0	0	0	0	0	0
	G17	50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	50
	G18	50	0	0	0	0	0	0	50	0	0	0	0	0	0	0	0	0	0	0	0	0
	G19	50	0	0	0	0	0	0	0	0	0	50	0	0	0	0	0	0	0	0	0	0
	G20	50	0	0	0	0	0	0	0	0	50	0	0	0	0	0	0	0	0	0	0	0
f_{20}	G01	1000	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50

Table 2: The experimental results of the RDG (with $\alpha = 10^{-12}$ or $\alpha = 10^{-10}$), DG2, and FII (with $\epsilon = 10^{-2}$ or $\alpha = 10^{-12}$) methods when used to decompose the CEC'2010 and CEC'2013 benchmark problems. "DA" is the decomposition accuracy; "FEs" is the function evaluations used; "ϵ" is the threshold. DG2 is a non-parametric method.

Bench-	Func	RDG ($\alpha = 10^{-12}$)			RDG ($\alpha = 10^{-10}$)			DG2		FII ($\epsilon = 10^{-2}$)		FII ($\alpha = 10^{-12}$)	
marks	Num	DA	FEs	ϵ	DA	FEs	ϵ	DA	FEs	DA	FEs	DA	FEs
CEC'2010	f_1	–	3.00e+03	4.11e-01	–	3.00e+03	4.11e+01	–	5.00e+05	–	3.00e+03	–	3.01e+03
	f_2	–	3.00e+03	2.49e-08	–	3.00e+03	2.49e-06	–	5.00e+05	–	3.00e+03	–	3.01e+03
	f_3	–	6.00e+03	2.15e-11	–	6.02e+03	2.15e-09	–	5.00e+05	–	3.00e+03	–	4.01e+03
	f_4	100%	4.20e+03	1.03e+04	100%	4.20e+03	1.03e+06	100%	5.00e+05	100%	3.69e+03	100%	3.06e+03
	f_5	100%	4.15e+03	1.14e-03	100%	4.15e+03	1.14e-01	100%	5.00e+05	100%	3.05e+03	100%	3.06e+03
	f_6	100%	5.00e+04	2.13e-05	100%	5.06e+03	2.13e-03	100%	5.00e+05	100%	3.05e+03	100%	3.12e+05
	f_7	100%	4.23e+03	5.17e+00	100%	4.23e+03	5.17e+02	100%	5.00e+05	100%	3.05e+03	100%	3.06e+03
	f_8	100%	5.60e+03	2.62e+05	100%	5.60e+03	2.62e+07	100%	5.00e+05	100%	1.88e+04	100%	3.66e+03
	f_9	100%	1.40e+04	4.88e-01	100%	1.40e+04	4.88e+01	100%	5.00e+05	100%	8.01e+03	100%	8.02e+03
	f_{10}	100%	1.40e+04	2.52e-08	100%	1.40e+04	2.52e-06	100%	5.00e+05	100%	8.01e+03	100%	8.02e+03
	f_{11}	100%	1.36e+04	2.36e-10	100%	1.39e+04	2.36e-08	100%	5.00e+05	97.2%	9.59e+03	100%	1.40e+04
	f_{12}	100%	1.43e+04	4.26e-05	100%	1.43e+04	4.26e-03	100%	5.00e+05	100%	8.01e+03	100%	8.02e+03
	f_{13}	100%	2.92e+04	3.71e+00	100%	2.92e+04	3.71e+02	100%	5.00e+05	100%	9.61e+04	100%	9.61e+04
	f_{14}	100%	2.05e+04	4.15e-01	100%	2.05e+04	4.15e+01	100%	5.00e+05	100%	2.30e+04	100%	2.30e+04
	f_{15}	100%	2.05e+04	2.53e-08	100%	2.05e+04	2.53e-06	100%	5.00e+05	100%	2.30e+04	100%	2.30e+04
	f_{16}	100%	2.09e+04	4.30e-10	100%	2.09e+04	4.30e-08	100%	5.00e+05	96.1%	3.09e+04	100%	2.30e+04
	f_{17}	100%	2.07e+04	1.10e-04	100%	2.07e+04	1.10e-02	100%	5.00e+05	100%	2.30e+04	100%	2.30e+04
	f_{18}	100%	4.98e+04	8.19e+00	100%	4.98e+04	8.19e+02	100%	5.00e+05	100%	3.69e+05	100%	3.69e+05
	f_{19}	100%	6.00e+03	6.14e-04	100%	6.00e+03	6.14e-02	100%	5.00e+05	100%	4.00e+03	100%	4.01e+03
	f_{20}	100%	5.08e+04	8.53e+00	100%	5.08e+04	8.53e+02	100%	5.00e+05	100%	5.03e+05	100%	5.03e+05
CEC'2013	f_1	–	3.00e+03	4.20e-01	–	3.00e+03	4.20e-+01	–	5.00e+05	–	3.00e+03	–	3.01e+03
	f_2	–	3.00e+03	1.31e-07	–	3.00e+03	1.31e-05	–	5.00e+05	–	3.00e+03	–	3.01e+03
	f_3	–	6.00e+03	2.16e-11	–	6.05e+03	2.16e-09	–	5.00e+05	–	3.00e+03	–	4.01e+03
	f_4	100%	9.84e+03	7.22e+01	100%	9.84e+03	7.22e+03	100%	5.00e+05	100%	4.55e+03	100%	4.56e+03
	f_5	100%	1.01e+04	8.03e-05	51.6%	9.02e+03	9.08e-03	100%	5.00e+05	66.3%	4.16e+03	98.3%	4.97e+03
	f_6	100%	1.32e+04	1.07e-06	83.3%	8.55e+03	1.07e-04	100%	5.00e+05	59.6%	3.68e+03	99.3%	6.32e+03
	f_7	100%	9.82e+03	5.82e+05	100%	9.82e+03	5.82e+07	83.3%	5.00e+05	33.3%	7.52e+03	100%	5.06e+03
	f_8	80.0%	1.95e+04	1.20e+06	46.8%	1.54e+04	4.35e+08	78.5%	5.00e+05	16.9%	4.92e+03	48.2%	1.16e+04
	f_9	100%	1.92e+04	6.07e-03	97.3%	2.02e+04	6.07e-01	100%	5.00e+05	99.6%	2.11e+04	99.9%	2.11e+04
	f_{10}	82.7%	1.91e+04	9.80e-05	76.6%	1.96e+04	9.80e-03	100%	5.00e+05	75.8%	1.53e+04	87.8%	1.92e+04
	f_{11}	10.0%	1.06e+04	1.52e+06	83.3%	1.10e+04	1.52e+08	100%	5.00e+05	10.0%	4.77e+03	97.0%	2.04e+04
	f_{12}	100%	5.08e+04	8.57e+00	100%	5.08e+04	8.57e+02	100%	5.00e+05	100%	5.03e+05	100%	5.03e+05
	f_{13}	–	8.39e+03	1.83e+06	–	8.31e+03	1.83e+08	–	4.10e+05	–	4.38e+03	–	8.89e+03
	f_{14}	–	1.61e+04	5.45e+06	–	1.60e+04	5.45e+08	–	4.10e+05	–	4.29e+03	–	9.50e+03
	f_{15}	100%	6.16e+03	2.70e+06	100%	8.11e+03	2.70e+08	100%	5.00e+05	100%	4.00e+03	100%	4.69e+03
Average	-	-	1.47e+04	-	-	1.45e+04	-	-	4.95e+05	-	4.94e+04	-	5.91e+04

Table 3: The results of the proposed RDG method when embedded into the DECC framework to solve the CEC'2010 benchmark problems. The RDG method is compared with GDG, XDG, DG, DG2, FII, D (delta grouping), and RG methods. The best performances are highlighted in bold (Wilcoxon rank-sum tests (α=0.05) with Holm p-value correction).

Func	Stats	RDG	GDG	XDG	DG	DG2	FII	D	RG
f_1	Median	1.50e-01	3.78e-10	5.58e+02	5.58e+02	4.02e+00	1.50e-01	**0.00e+00**	6.06e-14
	Mean	2.07e+00	3.96e-10	1.37e+04	1.37e+04	8.46e+02	2.07e+00	1.35e-26	9.15e-14
	Std	6.76e+00	1.21e-10	4.12e+04	4.12e+04	3.98e+03	6.76e+00	4.75e-26	7.88e-14
f_2	Median	4.46e+03	5.02e+02	4.46e+03	4.46e+03	4.36e+03	4.46e+03	2.89e+02	**1.17e+02**
	Mean	4.41e+03	4.99e+02	4.44e+03	4.44e+03	4.42e+03	4.41e+03	2.89e+02	1.14e+02
	Std	1.68e+02	2.03e+01	1.70e+02	1.70e+02	1.75e+02	1.68e+02	2.24e+01	2.65e+01
f_3	Median	1.67e+01	1.66e+01	1.68e+01	1.68e+01	1.67e+01	1.67e+01	**1.21e-13**	1.79e+00
	Mean	1.66e+01	1.67e+01	1.68e+01	1.68e+01	1.67e+01	1.66e+01	1.23e-13	1.77e+00
	Std	3.05e-01	3.33e-01	3.36e-01	3.36e-01	3.34e-01	3.37e-01	5.24e-15	3.15e-01
f_4	Median	**6.10e+11**	6.03e+13	**7.16e+11**	5.14e+12	**7.97e+11**	7.44e+11	3.00e+12	1.18e+13
	Mean	6.74e+11	6.53e+13	7.92e+11	5.54e+12	8.16e+11	8.86e+11	3.36e+12	1.10e+13
	Std	3.19e+11	2.01e+13	2.11e+11	2.11e+12	3.41e+11	3.50e+11	1.37e+12	2.84e+12
f_5	Median	**1.32e+08**	3.69e+08	1.61e+08	1.63e+08	1.44e+08	**1.24e+08**	2.62e+08	2.26e+08
	Mean	1.28e+08	3.66e+08	1.63e+08	1.61e+08	1.44e+08	1.27e+08	2.58e+08	2.46e+08
	Std	1.92e+07	2.20e+07	2.33e+07	2.10e+07	2.13e+07	2.08e+07	6.83e+07	5.41e+07
f_6	Median	**1.64e+01**	3.70e+02	**1.62e+01**	**1.64e+01**	**1.53e+01**	**1.64e+01**	3.55e-09	4.95e+06
	Mean	1.63e+01	3.63e+02	1.63e+01	1.64e+01	1.51e+01	1.64e+01	2.84e+06	5.04e+06
	Std	3.70e-01	8.81e+01	3.55e-01	3.61e-01	5.71e-01	3.68e-01	1.42e+07	8.78e+05
f_7	Median	**2.46e+00**	1.64e+10	4.32e+02	8.74e+03	2.33e+01	**8.84e+00**	3.28e+08	4.40e+06
	Mean	2.16e+01	1.71e+10	8.00e+02	1.51e+04	3.36e+02	6.76e+01	3.31e+08	5.13e+06
	Std	7.57e+01	3.17e+09	9.59e+02	1.37e+04	1.01e+03	1.28e+02	1.48e+08	3.70e+06
f_8	Median	**3.06e+00**	4.81e+07	1.59e+01	1.82e+07	5.95e+01	3.37e+07	8.31e+07	8.71e+07
	Mean	1.59e+05	8.82e+07	7.97e+05	2.73e+07	3.19e+05	4.16e+07	1.06e+08	7.34e+07
	Std	7.97e+05	6.85e+07	1.63e+06	2.19e+07	1.10e+06	2.17e+07	8.10e+07	3.17e+07
f_9	Median	**4.65e+07**	3.98e+08	1.10e+08	5.78e+07	6.63e+07	**4.53e+07**	6.30e+07	2.44e+08
	Mean	4.70e+07	4.03e+08	1.12e+08	5.85e+07	6.80e+07	4.64e+07	6.27e+07	2.42e+08
	Std	5.22e+06	2.89e+07	1.17e+07	7.43e+06	9.73e+06	4.92e+06	5.74e+06	2.68e+07
f_{10}	Median	4.33e+03	**3.44e+03**	5.27e+03	4.48e+03	4.64e+03	4.35e+03	1.31e+04	9.48e+03
	Mean	4.33e+03	3.43e+03	5.30e+03	4.50e+03	4.67e+03	4.34e+03	1.31e+04	9.29e+03
	Std	1.39e+02	5.34e+01	1.85e+02	1.17e+02	1.39e+02	1.40e+02	2.27e+02	1.30e+03
f_{11}	Median	1.01e+01	1.07e+01	1.07e+01	1.05e+01	1.03e+01	1.17e+01	**1.49e-13**	2.53e+01
	Mean	9.96e+00	1.04e+01	1.08e+01	1.04e+01	1.04e+01	1.16e+01	1.52e-13	2.51e+01
	Std	7.85e-01	1.05e+00	1.08e+00	1.09e+00	1.05e+00	1.16e+00	1.03e-14	1.46e+00
f_{12}	Median	**1.39e+03**	1.34e+05	1.16e+04	2.36e+03	3.94e+03	**1.36e+03**	4.33e+06	4.50e+04
	Mean	1.53e+03	1.34e+05	1.21e+04	2.38e+03	4.05e+03	1.62e+03	4.36e+06	4.47e+04
	Std	4.66e+02	6.92e+03	2.12e+03	3.01e+02	5.86e+02	6.77e+02	2.24e+05	5.11e+03
f_{13}	Median	**6.80e+02**	9.16e+02	2.98e+03	4.50e+03	1.28e+03	**7.42e+02**	1.03e+03	3.13e+03
	Mean	7.41e+02	9.40e+02	3.65e+03	5.22e+03	1.42e+03	8.86e+02	1.21e+03	4.00e+03
	Std	2.57e+02	2.00e+02	1.80e+03	2.93e+03	6.50e+02	5.34e+02	5.01e+02	2.53e+03
f_{14}	Median	3.48e+08	4.46e+08	5.82e+08	3.32e+08	4.61e+08	3.45e+08	**2.02e+08**	5.89e+08
	Mean	3.47e+08	4.53e+08	5.83e+08	3.42e+08	4.61e+08	3.50e+08	2.02e+08	5.86e+08
	Std	2.31e+07	3.98e+07	3.29e+07	2.30e+07	2.58e+07	2.15e+07	1.54e+07	4.45e+07
f_{15}	Median	**5.83e+03**	6.09e+03	6.35e+03	**5.84e+03**	6.11e+03	**5.86e+03**	1.58e+04	6.64e+03
	Mean	5.84e+03	6.09e+03	6.35e+03	5.84e+03	6.10e+03	5.85e+03	1.59e+04	8.61e+03
	Std	1.01e+02	8.91e+01	8.19e+01	7.48e+01	8.84e+01	7.60e+01	2.37e+02	3.23e+03
f_{16}	Median	**2.66e-13**	5.30e-11	1.73e-08	7.25e-13	5.31e-11	4.76e-13	2.20e-13	7.89e+01
	Mean	2.67e-13	5.47e-11	1.77e-08	7.42e-13	5.41e-11	4.67e-13	9.40e-02	7.77e+01
	Std	9.81e-15	5.61e-12	1.43e-09	6.10e-14	4.83e-12	3.17e-14	3.36e-01	1.47e+01
f_{17}	Median	**4.08e+04**	7.31e+04	1.28e+05	**4.11e+04**	7.25e+04	**4.14e+04**	7.49e+06	1.78e+05
	Mean	4.08e+04	7.43e+04	1.29e+05	4.07e+04	7.31e+04	4.07e+04	7.57e+06	1.76e+05
	Std	2.56e+03	4.37e+03	7.39e+03	2.49e+03	4.28e+03	3.08e+03	3.77e+05	1.02e+04
f_{18}	Median	**1.21e+03**	1.26e+03	1.38e+03	1.50e+10	1.28e+03	**1.23e+03**	1.81e+03	2.35e+04
	Mean	1.19e+03	1.28e+03	1.40e+03	1.48e+10	1.32e+03	1.21e+03	1.81e+03	2.35e+04
	Std	1.69e+02	1.57e+02	1.64e+02	2.34e+09	1.64e+02	1.47e+02	0.00e+00	0.00e+00
f_{19}	Median	1.73e+06	1.87e+06	1.73e+06	1.72e+06	1.86e+06	1.70e+06	1.85e+07	**7.88e+05**
	Mean	1.73e+06	1.88e+06	1.73e+06	1.72e+06	1.85e+06	1.72e+06	1.88e+07	7.74e+05
	Std	7.52e+04	7.34e+04	1.02e+05	9.27e+04	8.50e+04	1.06e+05	1.75e+06	3.94e+04
f_{20}	Median	4.09e+03	6.23e+03	3.64e+04	6.50e+10	6.25e+03	5.52e+03	**1.16e+03**	3.36e+03
	Mean	5.05e+03	2.85e+04	2.07e+05	6.55e+10	2.21e+04	9.83e+03	1.15e+03	3.39e+03
	Std	3.29e+03	7.50e+04	5.08e+05	8.03e+09	7.13e+04	1.13e+04	8.39e+01	3.04e+02

Table 4: The results of the proposed RDG method when embedded into the DECC framework to solve the CEC'2013 benchmark problems. The RDG method is compared with GDG, XDG, DG, DG2, FII, D (delta grouping), and RG methods. The best performances are highlighted in bold (Wilcoxon rank-sum tests (α=0.05) with Holm p-value correction).

Func	Stats	RDG	GDG	XDG	DG	DG2	FII	D	RG
f_1	Median	4.75e-01	5.15e-10	4.84e+02	4.84e+02	1.12e+01	4.75e-01	**0.00e+00**	1.32e-11
	Mean	3.83e+00	5.21e-10	1.31e+05	1.31e+05	1.56e+02	3.83e+00	4.31e-28	2.59e-11
	Std	6.38e+00	9.61e-11	5.40e+05	5.40e+05	4.78e+02	6.38e+00	1.46e-27	3.83e-11
f_2	Median	1.25e+04	4.61e+02	1.27e+04	1.27e+04	1.23e+04	1.25e+04	2.95e+02	**8.24e+01**
	Mean	1.26e+04	4.62e+02	1.27e+04	1.27e+04	1.25e+04	1.26e+04	2.94e+02	8.53e+01
	Std	5.78e+02	2.23e+01	6.62e+02	6.62e+02	5.44e+02	5.78e+02	1.83e+01	2.71e+01
f_3	Median	2.14e+01	2.14e+01	2.14e+01	2.14e+01	2.14e+01	2.14e+01	2.07e+01	**2.01e+01**
	Mean	2.14e+01	2.14e+01	2.14e+01	2.14e+01	2.14e+01	2.14e+01	2.07e+01	2.01e+01
	Std	1.34e-02	1.73e-02	1.32e-02	1.32e-02	1.30e-02	1.46e-02	2.22e-02	3.11e-03
f_4	Median	4.29e+10	2.85e+11	**8.78e+09**	8.02e+10	5.92e+10	4.42e+10	1.88e+10	8.49e+10
	Mean	4.17e+10	2.83e+11	8.73e+09	8.62e+10	5.79e+10	4.12e+10	1.86e+10	9.00e+10
	Std	1.51e+10	9.40e+10	2.13e+09	5.01e+10	2.21e+10	1.15e+10	8.94e+09	3.64e+10
f_5	Median	**5.09e+06**	7.72e+06	**4.78e+06**	**4.78e+06**	5.37e+06	5.39e+06	7.81e+06	8.61e+06
	Mean	5.09e+06	7.57e+06	4.89e+06	4.89e+06	5.37e+06	5.33e+06	7.65e+06	8.28e+06
	Std	4.81e+05	4.17e+05	5.80e+05	5.80e+05	4.90e+05	3.48e+05	1.93e+06	1.32e+06
f_6	Median	1.06e+06	1.06e+06	1.06e+06	1.06e+06	1.06e+06	1.06e+06	1.06e+06	1.06e+06
	Mean	1.06e+06	1.06e+06	1.06e+06	1.06e+06	1.06e+06	1.06e+06	1.06e+06	1.06e+06
	Std	1.10e+03	1.48e+03	1.62e+03	1.27e+03	1.60e+03	1.09e+03	2.10e+03	1.44e+03
f_7	Median	5.41e+07	2.29e+09	**1.51e+07**	3.88e+08	4.30e+07	1.43e+08	3.33e+09	2.83e+08
	Mean	6.42e+07	2.44e+09	1.57e+07	4.77e+08	4.37e+07	1.57e+08	3.45e+09	3.54e+08
	Std	2.97e+07	7.77e+08	5.46e+06	2.59e+08	1.73e+07	6.53e+07	1.33e+09	2.36e+08
f_8	Median	4.74e+15	7.87e+15	**2.39e+14**	3.74e+15	5.25e+15	7.73e+14	7.94e+14	2.51e+15
	Mean	5.04e+15	8.08e+15	3.11e+14	4.29e+15	5.20e+15	9.56e+14	9.01e+14	2.90e+15
	Std	1.86e+15	3.34e+15	1.86e+14	2.48e+15	1.59e+15	6.87e+14	3.43e+14	1.32e+15
f_9	Median	**4.73e+08**	4.99e+08	5.14e+08	**4.89e+08**	5.11e+08	4.98e+08	6.16e+08	5.68e+08
	Mean	4.78e+08	5.00e+08	5.08e+08	4.87e+08	5.13e+08	4.93e+08	5.90e+08	5.95e+08
	Std	2.43e+07	3.53e+07	3.54e+07	2.42e+07	3.58e+07	3.03e+07	7.72e+07	1.37e+08
f_{10}	Median	9.45e+07	9.47e+07	9.45e+07	9.46e+07	9.46e+07	9.45e+07	9.33e+07	**9.29e+07**
	Mean	9.45e+07	9.46e+07	9.46e+07	9.45e+07	9.46e+07	9.44e+07	9.33e+07	9.29e+07
	Std	2.61e+05	3.67e+05	2.14e+05	2.63e+05	2.65e+05	2.94e+05	4.48e+05	5.89e+05
f_{11}	Median	**5.18e+08**	7.27e+08	**4.83e+08**	2.60e+10	2.40e+09	**5.31e+08**	2.31e+10	5.19e+10
	Mean	5.56e+08	8.35e+08	5.23e+08	4.85e+10	4.95e+09	5.75e+08	6.02e+10	5.93e+10
	Std	1.97e+08	3.57e+08	1.30e+08	5.84e+10	6.88e+09	1.77e+08	7.18e+10	4.24e+10
f_{12}	Median	3.87e+03	5.63e+03	4.23e+04	1.65e+11	6.35e+03	6.00e+03	**1.26e+03**	3.36e+03
	Mean	4.02e+03	6.78e+03	5.80e+05	1.67e+11	6.97e+07	4.97e+04	1.25e+03	3.42e+03
	Std	6.64e+02	3.64e+03	1.70e+06	1.17e+10	3.48e+08	1.66e+05	1.09e+02	2.86e+02
f_{13}	Median	3.16e+09	1.47e+09	**1.11e+09**	1.86e+10	1.36e+09	**1.04e+09**	5.65e+10	5.56e+09
	Mean	3.06e+09	1.49e+09	1.14e+09	2.07e+10	1.45e+09	1.19e+09	5.58e+10	5.75e+09
	Std	6.68e+08	5.50e+08	4.00e+08	5.83e+09	3.54e+08	4.73e+08	9.85e+09	2.38e+09
f_{14}	Median	**2.50e+09**	6.71e+09	**3.13e+09**	1.66e+10	6.07e+09	**2.59e+09**	8.08e+11	6.35e+10
	Mean	2.87e+09	6.94e+09	4.27e+09	1.87e+10	6.64e+09	3.31e+09	7.96e+11	7.68e+10
	Std	1.73e+09	3.84e+09	3.19e+09	1.20e+10	3.31e+09	1.86e+09	2.75e+11	4.97e+10
f_{15}	Median	9.75e+06	1.05e+07	9.92e+06	9.28e+06	1.02e+07	8.97e+06	6.20e+07	**5.03e+06**
	Mean	1.10e+07	1.09e+07	1.01e+07	9.97e+06	1.04e+07	9.73e+06	6.26e+07	5.14e+06
	Std	3.76e+06	2.25e+06	1.80e+06	2.35e+06	2.45e+06	2.17e+06	7.92e+06	4.37e+05

Table 5: The results of the proposed RDG method when embedded into the CMAESCC framework to solve the CEC'2010 benchmark problems. The RDG method is compared with GDG, XDG, DG, DG2, FII, D (delta grouping), and RG methods. The best performances are highlighted in bold (Wilcoxon rank-sum tests (α=0.05) with Holm p-value correction).

Func	Stats	RDG	GDG	XDG	DG	DG2	FII	D	RG
f_1	Median	2.86e+05	**8.44e-21**	9.72e+05	9.72e+05	5.18e+05	2.86e+05	1.30e+06	1.56e-01
	Mean	2.84e+05	8.71e-21	9.71e+05	9.71e+05	5.23e+05	2.84e+05	1.34e+06	5.58e+01
	Std	2.28e+04	1.02e-21	5.92e+04	5.92e+04	4.31e+04	2.28e+04	4.23e+05	2.34e+02
f_2	Median	4.44e+03	**1.50e+03**	4.40e+03	4.40e+03	4.53e+03	4.44e+03	2.55e+03	2.57e+03
	Mean	4.43e+03	1.50e+03	4.43e+03	4.43e+03	4.51e+03	4.43e+03	2.52e+03	2.56e+03
	Std	1.77e+02	6.38e+01	2.03e+02	2.03e+02	1.73e+02	1.77e+02	9.58e+01	1.21e+02
f_3	Median	1.12e+00	1.09e+00	1.09e+00	1.09e+00	1.14e+00	1.09e+00	3.94e-13	**2.88e-13**
	Mean	1.06e+00	1.03e+00	9.48e-01	9.48e-01	1.05e+00	1.05e+00	3.96e-13	2.83e-13
	Std	3.49e-01	3.37e-01	4.37e-01	4.37e-01	4.06e-01	3.43e-01	1.73e-14	7.30e-15
f_4	Median	9.97e+05	2.40e+09	1.25e+11	8.57e+05	1.61e+06	**5.52e+05**	2.36e+11	1.44e+11
	Mean	1.01e+06	2.41e+09	1.24e+11	8.50e+05	1.60e+06	5.45e+05	2.58e+11	1.37e+11
	Std	9.37e+04	1.45e+09	1.09e+10	9.61e+04	1.33e+05	5.70e+04	1.12e+11	3.18e+10
f_5	Median	**9.05e+07**	1.04e+08	**9.27e+07**	**9.85e+07**	**9.45e+07**	**1.00e+08**	3.08e+08	2.79e+08
	Mean	9.52e+07	1.06e+08	9.33e+07	9.66e+07	9.13e+07	9.88e+07	3.27e+08	2.91e+08
	Std	2.23e+07	2.04e+07	1.78e+07	1.82e+07	2.07e+07	2.28e+07	9.26e+07	7.16e+07
f_6	Median	1.05e+00	**6.04e-08**	1.16e+00	1.03e+00	1.58e+00	1.06e+00	3.91e+06	2.58e+06
	Mean	9.17e-01	6.10e-08	1.12e+00	9.09e-01	1.57e+00	9.50e-01	3.93e+06	2.81e+06
	Std	4.23e-01	8.68e-09	1.10e-01	4.22e-01	9.62e-02	3.77e-01	9.55e+05	7.41e+05
f_7	Median	**7.41e-19**	3.05e-18	**7.41e-19**	4.66e+05	**7.59e-19**	**7.84e-19**	1.77e+05	4.18e+06
	Mean	7.41e-19	2.81e-18	7.47e-19	4.55e+05	7.59e-19	7.56e-19	1.87e+05	4.20e+06
	Std	8.35e-20	7.86e-19	9.28e-20	9.15e+04	9.22e-20	8.11e-20	6.85e+04	2.16e+05
f_8	Median	**2.16e-17**	2.11e+07	**1.90e-17**	6.26e-01	**2.12e-17**	1.57e+06	7.70e+06	9.12e+06
	Mean	7.97e+05	3.20e+07	4.78e+05	6.38e+05	4.78e+05	1.52e+06	2.36e+07	2.46e+07
	Std	1.63e+06	3.75e+07	1.32e+06	1.49e+06	1.32e+06	2.95e+05	2.97e+07	2.44e+07
f_9	Median	4.75e+06	**6.64e+02**	1.00e+07	5.82e+06	6.62e+06	4.62e+06	2.97e+07	6.57e+06
	Mean	4.82e+06	7.62e+02	1.02e+07	5.82e+06	6.62e+06	4.64e+06	2.91e+07	6.67e+06
	Std	5.25e+05	3.19e+02	9.80e+05	5.49e+05	4.33e+05	3.95e+05	3.94e+06	8.65e+05
f_{10}	Median	2.89e+03	**1.73e+03**	2.84e+03	2.76e+03	2.84e+03	2.83e+03	4.01e+03	4.17e+03
	Mean	2.88e+03	1.72e+03	2.82e+03	2.81e+03	2.84e+03	2.84e+03	4.03e+03	4.16e+03
	Std	1.29e+02	7.73e+01	1.22e+02	1.30e+02	1.38e+02	1.27e+02	1.93e+02	1.82e+02
f_{11}	Median	**1.51e-12**	**1.52e-12**	**1.52e-12**	2.10e+01	**1.52e-12**	2.97e+01	1.15e+02	1.17e+02
	Mean	3.58e-02	7.64e-02	7.70e-02	2.09e+01	3.52e-02	2.99e+01	1.12e+02	1.16e+02
	Std	1.79e-01	2.65e-01	2.67e-01	3.83e-01	1.76e-01	2.19e+00	1.49e+01	2.02e+01
f_{12}	Median	4.31e-22	**5.51e-24**	4.37e-22	4.09e-22	4.38e-22	4.27e-22	4.12e+04	3.64e-04
	Mean	4.23e-22	5.56e-24	4.37e-22	3.86e-22	4.26e-22	4.17e-22	3.87e+04	3.83e-04
	Std	8.39e-23	3.77e-25	3.60e-23	1.34e-22	8.96e-23	8.20e-23	1.05e+04	1.59e-04
f_{13}	Median	**3.99e+00**	1.42e+02	**3.99e+00**	6.80e+02	**3.99e+00**	**3.99e+00**	1.03e+03	3.08e+02
	Mean	5.90e+00	1.85e+02	5.26e+00	6.98e+02	5.90e+00	7.02e+00	1.32e+03	3.62e+02
	Std	4.01e+00	8.07e+01	3.77e+00	2.92e+02	4.32e+00	4.92e+00	6.99e+02	1.86e+02
f_{14}	Median	**3.91e-20**	2.03e-19	4.73e-01	4.07e-20	1.89e-19	4.07e-20	6.91e+07	1.85e+07
	Mean	3.91e-20	2.04e-19	1.34e+00	4.06e-20	1.98e-19	4.04e-20	7.11e+07	1.85e+07
	Std	2.12e-21	4.17e-20	2.42e+00	1.89e-21	3.35e-20	1.44e-21	1.01e+07	1.88e+06
f_{15}	Median	**1.93e+03**	**1.93e+03**	**1.93e+03**	**1.91e+03**	**1.92e+03**	**1.94e+03**	4.34e+03	4.29e+03
	Mean	1.95e+03	1.92e+03	1.91e+03	1.93e+03	1.92e+03	1.94e+03	4.35e+03	4.37e+03
	Std	1.11e+02	9.80e+01	9.95e+01	9.41e+01	6.82e+01	6.68e+01	2.17e+02	2.97e+02
f_{16}	Median	**8.42e-13**	8.70e-13	9.45e-13	**8.53e-13**	8.70e-13	**8.14e-13**	2.19e+02	2.23e+02
	Mean	8.44e-13	8.72e-13	9.43e-13	8.48e-13	8.73e-13	8.12e-13	2.12e+02	2.16e+02
	Std	2.10e-14	2.11e-14	2.73e-14	2.16e-14	2.34e-14	2.15e-14	2.00e+01	2.96e+01
f_{17}	Median	**6.90e-24**	7.21e-24	8.29e-24	**6.95e-24**	7.33e-24	**6.94e-24**	1.18e+05	9.77e+00
	Mean	6.91e-24	7.29e-24	8.29e-24	6.93e-24	7.36e-24	6.97e-24	1.19e+05	9.95e+00
	Std	2.06e-25	3.06e-25	3.14e-25	2.38e-25	2.48e-25	2.53e-25	1.31e+04	1.71e+00
f_{18}	Median	**1.55e+01**	3.01e+01	1.78e+02	4.33e+03	3.22e+01	2.33e+01	2.50e+03	1.27e+03
	Mean	1.50e+01	4.97e+01	2.00e+02	7.35e+04	4.33e+01	3.27e+01	2.47e+03	1.37e+03
	Std	7.20e+00	4.67e+01	7.68e+01	1.14e+05	3.01e+01	2.53e+01	8.60e+02	4.26e+02
f_{19}	Median	**5.64e+03**	2.00e+04	**5.30e+03**	**5.33e+03**	1.99e+04	**5.30e+03**	6.29e+06	9.69e+06
	Mean	5.47e+03	1.98e+04	5.29e+03	5.23e+03	2.00e+04	5.29e+03	6.45e+06	9.79e+06
	Std	7.08e+02	1.89e+03	6.35e+02	7.31e+02	2.38e+03	6.35e+02	8.17e+05	6.80e+05
f_{20}	Median	**8.55e+02**	**8.29e+02**	8.65e+02	1.09e+03	**8.30e+02**	**8.27e+02**	9.75e+02	9.67e+02
	Mean	8.27e+02	8.34e+02	8.61e+02	1.12e+03	8.36e+02	8.24e+02	9.90e+02	9.72e+02
	Std	6.35e+01	5.79e+01	4.79e+01	1.05e+02	5.07e+01	5.32e+01	2.68e+01	1.20e+01

Table 6: The results of the proposed RDG method when embedded into the CMAESCC framework to solve the CEC'2013 benchmark problems. The RDG method is compared with GDG, XDG, DG, DG2, FII, D (delta grouping), and RG methods. The best performances are highlighted in bold (Wilcoxon rank-sum tests (α=0.05) with Holm p-value correction).

Func	Stats	RDG	GDG	XDG	DG	DG2	FII	D	RG
f_1	Median	2.85e+05	**1.02e-20**	1.02e+06	1.02e+06	5.48e+05	2.85e+05	1.61e+06	8.26e+02
	Mean	2.90e+05	1.04e-20	1.04e+06	1.04e+06	5.52e+05	2.90e+05	1.54e+06	1.64e+03
	Std	3.28e+04	9.90e-22	1.13e+05	1.13e+05	5.88e+04	3.28e+04	5.19e+05	1.94e+03
f_2	Median	4.67e+03	**1.55e+03**	4.72e+03	4.72e+03	4.69e+03	4.67e+03	2.64e+03	2.72e+03
	Mean	4.69e+03	1.54e+03	4.74e+03	4.74e+03	4.69e+03	4.69e+03	2.66e+03	2.69e+03
	Std	1.78e+02	7.52e+01	2.24e+02	2.24e+02	1.81e+02	1.78e+02	1.70e+02	1.02e+02
f_3	Median	2.04e+01	2.04e+01	2.05e+01	2.05e+01	2.04e+01	2.04e+01	2.02e+01	**2.00e+01**
	Mean	2.04e+01	2.04e+01	2.05e+01	2.05e+01	2.04e+01	2.04e+01	2.02e+01	2.00e+01
	Std	4.96e-02	4.28e-02	5.25e-02	5.25e-02	5.21e-02	5.29e-02	1.90e-02	7.86e-03
f_4	Median	5.81e+06	**6.35e+04**	3.55e+09	1.45e+10	8.43e+06	5.91e+06	2.37e+09	8.59e+08
	Mean	5.83e+06	7.31e+04	3.51e+09	1.49e+10	8.52e+06	5.88e+06	2.45e+09	9.48e+08
	Std	6.32e+05	3.71e+04	3.02e+08	2.21e+09	8.54e+05	5.44e+05	9.95e+08	3.78e+08
f_5	Median	2.35e+06	2.23e+06	**1.60e+06**	**1.60e+06**	2.17e+06	2.15e+06	5.90e+06	5.63e+06
	Mean	2.40e+06	2.23e+06	1.59e+06	1.59e+06	2.19e+06	2.22e+06	5.82e+06	5.66e+06
	Std	4.36e+05	4.24e+05	2.38e+05	2.38e+05	3.51e+05	3.88e+05	1.10e+06	1.28e+06
f_6	Median	**9.96e+05**	**9.96e+05**	**9.96e+05**	**9.96e+05**	**9.96e+05**	**9.96e+05**	1.06e+06	1.06e+06
	Mean	9.96e+05	9.96e+05	9.96e+05	9.96e+05	9.96e+05	9.96e+05	1.06e+06	1.06e+06
	Std	1.48e+02	1.70e+03	8.83e+00	5.57e+02	3.31e+02	5.93e+01	1.49e+03	1.29e+03
f_7	Median	**2.94e-20**	3.61e+07	9.22e+05	1.36e+06	1.00e+03	1.56e+05	6.27e+06	1.31e+06
	Mean	8.12e-17	3.73e+07	9.20e+05	1.40e+06	1.05e+03	1.53e+05	6.90e+06	1.33e+06
	Std	2.17e-16	1.30e+07	7.47e+04	1.81e+05	2.79e+02	2.53e+04	3.55e+06	1.35e+05
f_8	Median	**8.26e+06**	1.14e+08	2.67e+13	5.68e+13	3.57e+07	5.57e+13	5.47e+13	3.98e+13
	Mean	8.51e+06	1.28e+08	2.73e+13	5.58e+13	3.85e+07	5.73e+13	5.35e+13	4.25e+13
	Std	2.92e+06	3.52e+07	8.01e+12	2.04e+13	1.09e+07	1.28e+13	1.65e+13	1.84e+13
f_9	Median	**1.58e+08**	**1.58e+08**	**1.51e+08**	**1.61e+08**	**1.52e+08**	**1.68e+08**	4.41e+08	4.74e+08
	Mean	1.65e+08	1.67e+08	1.66e+08	1.56e+08	1.51e+08	1.71e+08	4.54e+08	4.96e+08
	Std	4.16e+07	3.88e+07	3.49e+07	2.75e+07	2.87e+07	3.29e+07	9.83e+07	1.17e+08
f_{10}	Median	**9.05e+07**	9.06e+07	**9.05e+07**	9.05e+07	**9.05e+07**	**9.05e+07**	9.32e+07	9.29e+07
	Mean	9.10e+07	9.11e+07	9.07e+07	9.14e+07	9.13e+07	9.07e+07	9.33e+07	9.30e+07
	Std	1.29e+06	1.20e+06	3.09e+05	1.64e+06	1.51e+06	8.45e+05	3.65e+05	5.71e+05
f_{11}	Median	1.68e+07	2.57e+07	1.71e+07	2.90e+08	**1.56e+05**	1.72e+07	1.32e+08	1.32e+08
	Mean	1.67e+07	2.53e+07	1.73e+07	4.67e+08	2.47e+05	1.67e+07	1.63e+08	1.63e+08
	Std	1.62e+06	2.69e+06	1.62e+06	3.29e+08	2.37e+05	1.67e+06	9.02e+07	9.02e+07
f_{12}	Median	**1.01e+03**	1.02e+03	1.03e+03	1.24e+03	1.02e+03	1.02e+03	1.03e+03	1.03e+03
	Mean	9.81e+02	1.00e+03	1.02e+03	1.25e+03	1.01e+03	1.02e+03	1.05e+03	1.03e+03
	Std	7.30e+01	3.91e+01	4.67e+01	1.36e+02	5.81e+01	4.52e+01	2.43e+01	1.76e+01
f_{13}	Median	2.49e+06	2.29e+06	**1.47e+06**	3.40e+07	2.28e+06	**1.59e+06**	1.93e+09	1.31e+08
	Mean	2.47e+06	2.36e+06	1.53e+06	3.40e+07	2.43e+06	1.55e+06	1.96e+09	1.49e+08
	Std	3.83e+05	3.38e+05	2.00e+05	1.10e+07	3.70e+05	1.85e+05	1.02e+09	7.64e+07
f_{14}	Median	2.74e+07	3.60e+07	2.75e+07	**6.22e+06**	3.66e+07	2.73e+07	1.58e+09	1.85e+08
	Mean	2.77e+07	3.63e+07	2.81e+07	7.26e+06	3.59e+07	2.74e+07	2.79e+09	1.84e+08
	Std	1.80e+06	3.18e+06	2.25e+06	2.12e+06	2.85e+06	2.40e+06	3.47e+09	3.22e+07
f_{15}	Median	**2.19e+06**	3.04e+06	**2.36e+06**	**2.21e+06**	2.93e+06	**2.36e+06**	1.72e+07	3.68e+06
	Mean	2.19e+06	3.05e+06	2.33e+06	2.25e+06	3.02e+06	2.33e+06	1.69e+07	6.69e+06
	Std	2.28e+05	3.35e+05	2.92e+05	1.55e+05	3.30e+05	2.92e+05	1.80e+06	7.49e+06