Cooperative Co-evolution with Online Optimizer Selection for Large-Scale Optimization

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Overview,

- Introduction
- 2 Background and Related Work
- 3 Cooperative Co-evolution with Online Optimizer Selection
- Experimental Methodology and Results
- Conclusion

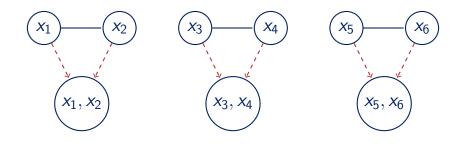
Introduction: Large-Scale Continuous Optimization

Large-scale (High-dimensional) Continuous Optimization Problems are challenging to solve:

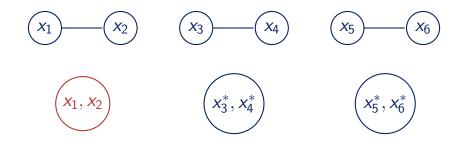
- Search space increases exponentially.
- Problem complexity increases greatly.
- The running time of some evolutionary algorithms increases significantly.



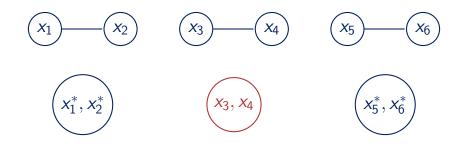
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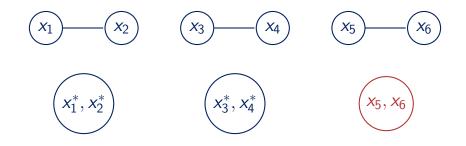
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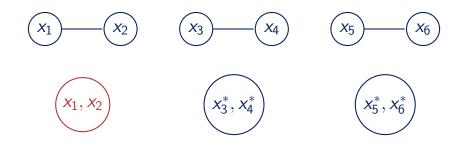
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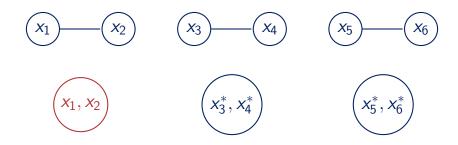
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Limitation of CC: Inefficient to solve problems with imbalanced components (contributing differently to the overall fitness values).

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Components:

$$\begin{pmatrix} x_1, x_2 \end{pmatrix}$$

$$(x_3, x_4)$$

$$\left(x_5, x_6\right)$$

²Yang M, Omidvar M N, Li C, et al. Efficient resource allocation in cooperative co-evolution for large-scale global optimization[J]. IEEE Transactions on Evolutionary Computation, 2017, 21(4): 493-505.

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 U_1 :

 U_2 :

 U_3 :

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 U_3 :

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 U_1



8

 U_2 :

 U_3 : (

$$t = 0$$
 $t = 1$

$$U_i = (\hat{U}_i + \hat{y}_b - y_b)/2. \tag{1}$$

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$$t = 0$$

t = 0 t = 1 t = 2 t = 3

$$= 2$$

$$t=3$$

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$$\left(x_5, x_6\right)$$

$$t = 0$$

$$t = 0$$
 $t = 1$ $t = 2$ $t = 3$ $t = 4$

$$= 2$$

$$= 3$$

$$t = 4$$

$$U_i = (U_i + \hat{y}_b - y_b)/2$$

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$$U_1$$
 :

$$U_3$$

$$t = 0$$

$$t = 1$$

$$t = 0$$
 $t = 1$ $t = 2$ $t = 3$ $t = 4$ $t = 5$

$$= 3$$

$$= 4$$

$$=$$
 5

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Algorithms:

 (a_1)

 a_2

Components:

 (c_1)



Algorithms:

 a_1

 a_2

Components:

 (c_1)

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 U_{a_1,c_1} :

 U_{a_1,c_2} :

 U_{a_2,c_1} :

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Algorithms:

 a_1

 a_2

Components:

 (c_1)

 (c_2)

 $U_{a_1,c_1}:$ 0

 $U_{a_1,c_2}:$ $\left(0\right)$

 $U_{a_2,c_1}: \left(0\right)$

 U_{a_2,c_2} : 0

t = 0

Algorithms:

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(8)

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 $U_{a_2,c_1}:$ $\left(0\right)$

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t = 0 t = 1

 $U_{a_i,c_j} = \frac{\hat{U}_{a_i,c_j} + (\hat{y}_b - y_b)/\hat{y}_b}{2}.$

Algorithms:

 a_1

Components:

 (c_1)

 (c_2)

 $U_{a_1,c_1}: (0)$

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 $U_{a_1,c_2}:$ 0

4

 $U_{a_2,c_1}:$ $\left(0\right)$

 $U_{a_2,c_2}:$ (0)

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 c_2

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Components:

 U_{a_1,c_1} :

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$$t = 0$$
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Algorithms:

Components:

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Components:



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 $\left(0\right)$

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Components:





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$$t =$$

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1 using an array: $\Theta(|\mathbb{A}||\mathbb{C}|T)$.

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t = i $\begin{vmatrix} a_1; c_1; 2 & a_1; c_2; 1 & a_1; c_3; 9 & a_2; c_1; 3 & a_2; c_2; 5 & a_2; c_3; 7 \end{vmatrix}$

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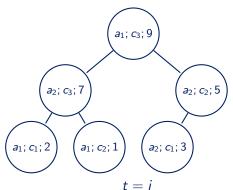
$$t = i t = i a1; c1; 2 | a1; c2; 1 | a1; c3; 9 | a2; c1; 3 | a2; c2; 5 | a2; c3; 7 t = i + 1 | a1; c1; 2 | a1; c2; 1 | a1; c3; 6 | a2; c1; 3 | a2; c2; 5 | a2; c3; 7$$

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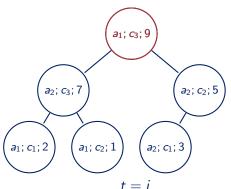
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t = i	$a_1; c_1; 2$	$a_1; c_2; 1$	<i>a</i> ₁ ; <i>c</i> ₃ ; 9	<i>a</i> ₂ ; <i>c</i> ₁ ; 3	<i>a</i> ₂ ; <i>c</i> ₂ ; 5	<i>a</i> ₂ ; <i>c</i> ₃ ; 7
t = i + 1	<i>a</i> ₁ ; <i>c</i> ₁ ; 2	$a_1; c_2; 1$	<i>a</i> ₁ ; <i>c</i> ₃ ; 6	<i>a</i> ₂ ; <i>c</i> ₁ ; 3	<i>a</i> ₂ ; <i>c</i> ₂ ; 5	<i>a</i> ₂ ; <i>c</i> ₃ ; 7

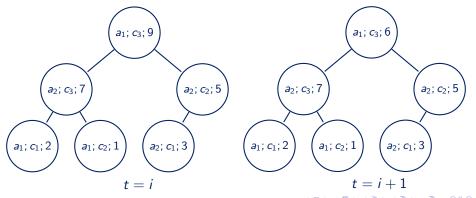


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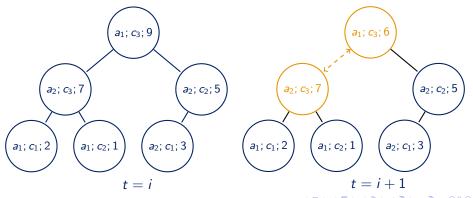
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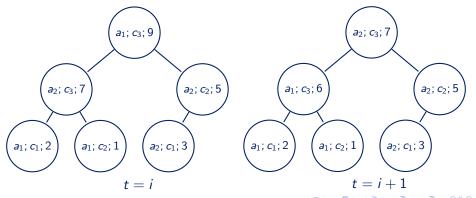


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- Decomposition method: Recursive Differential Grouping³ which uses $O(n \log(n))$ function evaluations in decomposition.

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- CCOS: uses Self-adaptive Differential Evolution with Neighborhood Search (SaNSDE) and Social-Learning Particle Swarm Optimization (SL-PSO) as the candidate optimizers.

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- CCDE: only uses SaNSDE as the optimizer.

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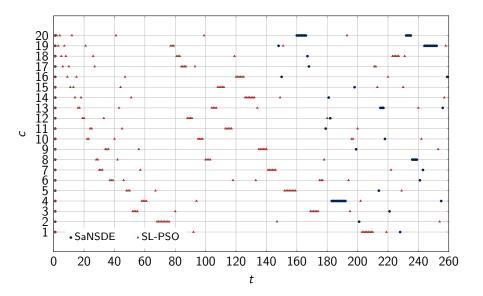
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- CCDE: only uses SaNSDE as the optimizer.
- CCPSO: only uses SL-PSO as the optimizer.
- Statistical test: Wilcoxon rank-sum test with 95% confidence interval.

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Experimental Results: Selection Details of CCOS (f_{16})



Experimental Results: Selection Ability of CCOS

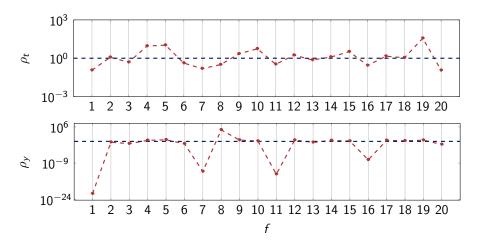
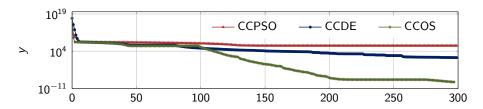
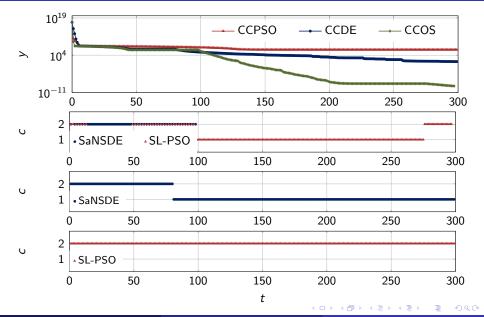


Figure: ρ_t : the ratio of the number of evolutionary cycles that DE and PSO were selected; ρ_y : the ratio of the solution quality generated by CCPSO and CCDE.

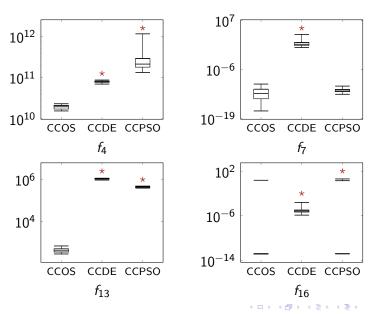
Experimental Results: Selection Ability of CCOS (f_8)



Experimental Results: Selection Ability of CCOS (f_8)



Experimental Results: Comparison with CCDE and CCPSO



Experimental Results: Comparison with State-of-the-arts

Table: The optimization results of CCOS, CSO, MOS and MA-SW-Chain when used to solve the CEC'2010 benchmark problems (Wilcoxon rank-sum tests).

Func	Stats	CCOS	CSO	MOS	MA-SW-Chain
f_4	median	1.96e+10	7.26e+11	4.94e+11	3.10e+11
	mean	1.95e+10	7.25e+11	5.16e+11	2.97e+11
	std	2.23e+09	1.23e+11	1.85e+11	6.19e+10
f ₇	median	3.94e-13	2.04e+04	2.27e+07	7.94e-03
	mean	9.15e-12	2.01e+04	3.54e+07	1.17e+02
	std	2.61e-11	3.86e+03	3.22e+07	2.37e+02
f ₁₃	median	3.85e+02	5.47e+02	3.19e+02	8.61e+02
	mean	4.10e+02	6.29e+02	3.32e+02	9.83e+02
	std	1.09e+02	2.32e+02	1.19e+02	5.66e+02
f ₁₆	median	1.49e-13	5.75e-08	3.97e+02	9.32e+01
	mean	1.88e-01	5.89e-08	3.96e+02	9.95e+01
	std	5.53e-01	5.61e-09	3.47e+00	1.53e+01

Conclusion

Proposed an online optimizer selection framework to select the best optimizer from a portfolio for each component when solving large-scale optimization problems using CC algorithm.

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- Experimentally demonstrated that the proposed CCOS algorithm was successful in selecting the best optimizer when solving the CEC'2010 benchmark problems.

Conclusion

- Proposed an online optimizer selection framework to select the best optimizer from a portfolio for each component when solving large-scale optimization problems using CC algorithm.
- Experimentally demonstrated that the proposed CCOS algorithm was successful in selecting the best optimizer when solving the CEC'2010 benchmark problems.
- Showed that CCOS could potentially generate statistically better solution quality than the default CC algorithm with no optimizer selection ability.

Thank You! & Questions?